Expectation and Duration at the Effective Lower Bound

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[1] The views expressed here do not represent those of the Chicago Fed or the Federal Reserve System.

Introduction

This paper studies

- the impact of **duration exposures** and **short-rate expectations**,
- in a structural, **equilibrium model** of the yield curve,
- with an **effective lower bound**.

The main interest is in analyzing the effects of alternative monetary policy tools at the ELB.

"Structural" part of the model:

- Risk-averse arbitrageurs
- Vayanos & Vila (2009); Greenwood & Vayanos (2014); King (2015)

ELB:

- Shadow-rate process
- Kim & Singleton (2012); Krippner (2012); Wu & Xia (2015)

Factor loadings change qualitatively and quantitatively by introducing the ELB.

Introduction

Some *prima facie* evidence that this is important...

Extend Greenwood-Vayanos regressions through 2015, allowing break in 2008.

	Independent variables						
	WAM of Treas. debt			1y yield			Adj. R^2
Dep. Var.	Pre-ELB	ELB	*Break t-stat*	Pre-ELB	ELB	*Break t-stat*	
5y yield	0.140 (0.095)	0.002 (0.101)	*-2.17*	0.842*** (0.050)	2.271*** (0.785)	*1.84*	0.951
10y yield	0.221* (0.121)	0.058 (0.116)	*-2.25*	0.736*** (0.060)	3.028** (1.203)	*1.92*	0.901
15y yield	0.261* (0.133)	0.110 (0.126)	*-2.05*	0.688*** (0.065)	2.966** (1.276)	*1.80*	0.870

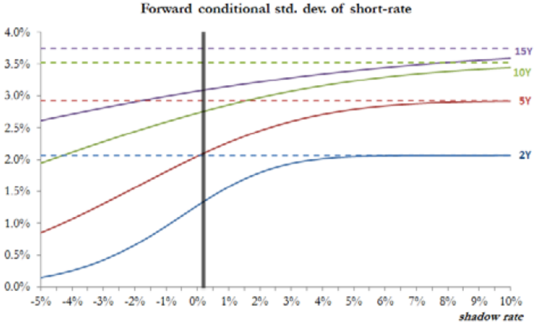
	Independent variables						
	WAM of Treas. debt			2y yield			Adj. R^2
Dep. Var.	Pre-ELB	ELB	*Break t-stat*	Pre-ELB	ELB	*Break t-stat*	
5y yield	0.102* (0.373)	-0.002 (0.060)	*-2.57*	0.901*** (0.032)	1.910*** (0.217)	*4.74*	0.981
10y yield	0.187** (0.094)	0.053 (0.088)	*-2.25*	0.794*** (0.048)	2.328*** (0.429)	*3.61*	0.942
15y yield	0.227** (0.109)	0.113 (0.108)	*-1.62*	0.746*** (0.056)	2.167*** (0.537)	*2.68*	0.915

Introduction

Why does this happen in the model?

term premium $\approx$ risk aversion $\times$ duration exposure $\times$ interest-rate vol

- ELB dampens interest-rate vol:



- Yields become less responsive to duration.
- Shadow rate induces changes in term premia.

Outline

- Illustrate basics in a one-factor model.
- Extend the model to allow for stochastic bond supply.
- Calibrate to long-run U.S. yield moments and solve it numerically.
- Show that it matches:
  - Conditional moments at the ELB.
  - The regression coefficients just presented.
  - Event-study evidence on QE.
- Look briefly at how the ELB affects factor loadings and other results.
- Use the model to examine the effectiveness alternative unconventional policies.
  - Feed the model shadow-rate and bond-supply shocks that resemble the Fed's actions.
  - Check the contribution of each.

Model setup - constant bond supply

Arbitrageurs solve

$$\max_{x_t(\tau)\forall\tau} \mathrm{E}_t\left[dW_t\right] - \frac{a}{2}\mathrm{var}_t\left[dW_t\right]$$

subject to

$$dW_t = \int_0^T x_t\left(\tau\right)\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}d\tau + \left(W_t - \int_0^T x_t\left(\tau\right)d\tau\right)r_t dt$$

where W_t is wealth, $x_t\left(\tau\right)$ is bond holdings at maturity τ, $P_t^{(\tau)}$ is the bond price at maturity τ, and r_t is the short rate.

The government supplies bonds ζ at all maturities. Equilibrium is determined by

$$x_t\left(\tau\right) = \zeta$$

for all τ.

Equilibrium

$$\mathrm{E}_t\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right] = r_t dt + a\int_0^T \zeta \operatorname{cov}_t\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}, \frac{dP_t^{(s)}}{P_t^{(s)}}\right] ds$$

Assume the shadow-rate process:

$$r_t = \max\left[\widehat{r}_t, b\right]$$

$$d\widehat{r}_t = \kappa(\mu - \widehat{r}_t)dt + \sigma dB_t$$

Then

$$\underbrace{\mathrm{E}_t\left[\frac{dP_t^{(\tau)}}{P_t^{(\tau)}}\right] - r_t dt}_{\text{risk premium}} = \zeta\left(a\sigma^2 A_t^{(\tau)}\int_0^T A_t^{(s)} ds\right)$$

where $A_t^{(\tau)}$ is the sensitivity of the τ-maturity price to $\widehat{r}_t$.

Factor loadings

If $b = -\infty$, the model is affine and

$$A_t^{(\tau)} = \int_0^{\tau} e^{-\kappa s} ds = \frac{1 - e^{-\kappa \tau}}{\kappa}$$

This is the Greenwood-Vayanos-Vila one-factor model.

At each maturity:

- Return volatility is constant.
- Risk premium is constant.
- Sensitivity to bond supply is constant.

Factor loadings

Affine case, $b = -\infty$:

$$A^{(\tau)} = \int_0^\tau e^{-\kappa s} ds$$

Shadow-rate case, $b > -\infty$:

$$A_t^{(\tau)} \approx \int_0^\tau e^{-\kappa s} \Phi_t^{(s)} ds$$

where $\Phi_t^{(s)} = \mathrm{Pr}_t \left[\widehat{r}_{t+s} > b \right]$.

Note:

- $A_t^{(\tau)}$ is strictly increasing in $\widehat{r}_t$.
- Lower $\widehat{r}_t$ means lower volatility, expected returns, and supply sensitivity.
- The affine GVV model is a limiting case that holds when the ELB never binds.
- The result is not exact because now term premia depend on $\widehat{r}_t$ too.

Stochastic bond supply

Now let there be a stochastic bond supply $s_t(\tau)$ at each maturity.

Following Greenwood et al. (2015), reduce bond supply to a single factor:

$$s_t(\tau) = \zeta + \left(1 - \frac{2\tau}{T}\right)\beta_t$$

$$\beta_t = \phi_\beta \beta_{t-1} + e_t^\beta \qquad e_t^\beta \sim Niid\left(0, \sigma_\beta\right)$$

Maturity distribution moves in a see-saw pattern in response to shocks to β_t.

(The shape of the distribution is not of major importance.)

Stochastic bond supply

The WAM of outstanding debt is

$$WAM_t \equiv v \frac{\int\limits_0^T \tau s_t(\tau)\, d\tau}{\int\limits_0^T s_t(\tau)_t\, d\tau} = vT(\frac{1}{2} - \frac{1}{6\zeta}\beta_t)$$

where v is the length of one period, in years.

Outstanding 10-year equivalents are

$$\%\Delta 10YE_t \equiv \frac{\frac{v}{10}\int\limits_0^T \tau s_t(\tau)\, d\tau}{\frac{v}{10}\int\limits_0^T \tau s_{t-1}(\tau)\, d\tau} = -\frac{\Delta\beta_{t+h}}{3\zeta - \beta_t}$$

Calibration and solution

Bond supply				Short rate				Risk aversion
T	κ_β	σ_β	ζ	μ	κ	σ	b	a
60	0.021	0.20	0.37	4.9%	0.019	0.77%	0.17%	0.15

- Using data since 1971, I match:
    - the annual autocorrelation of Treasury WAM
    - the unconditional mean and std. dev. of the 3M and 10Y yield
    - the unconditional correlation between the 3M and 10Y yield
    - the mean 3M yield during the ELB period
- Model is solved numerically using an iterative projection method.

Evidence on the model's fit

Short rate below 0.68%

	% of obs.	3m rate	Slopes (to 3m) 2Y	5Y	10Y	15Y
Conditional means						
Data	*16%*	*0.2%*	*0.3%*	*1.3%*	*2.5%*	*3.1%*
Shadow-rate model	15%	0.2%	0.4%	1.2%	2.4%	3.5%
Affine Model – base calibration	15%	-1.3%	0.7%	1.8%	3.4%	4.5%
Affine Model – recalibrated	10%	-0.9%	0.7%	1.8%	3.3%	4.5%
Conditional standard deviations						
Data		*0.1%*	*0.3%*	*0.6%*	*0.8%*	*0.8%*
Shadow-rate model		0.2%	0.3%	0.7%	1.1%	1.4%
Affine Model – base calibration		1.7%	0.3%	0.7%	1.3%	1.7%
Affine Model – recalibrated		1.5%	0.3%	0.7%	1.2%	1.5%

Short rate above 0.68%

	% of obs.	3m rate	Slopes (to 3m) 2Y	5Y	10Y	15Y
Conditional means						
Data	*84%*	*6.1%*	*0.5%*	*0.9%*	*1.3%*	*1.5%*
Shadow-rate model	85%	6.0%	0.3%	0.7%	1.3%	1.8%
Affine Model – base calibration	85%	6.0%	0.3%	0.7%	1.2%	1.7%
Affine Model – recalibrated	90%	5.9%	0.3%	0.7%	1.3%	1.8%
Conditional standard deviations						
Data		*3.1%*	*0.9%*	*1.3%*	*1.6%*	*1.7%*
Shadow-rate model		3.2%	0.3%	0.8%	1.5%	1.9%
Affine Model – base calibration		3.2%	0.3%	0.8%	1.5%	2.0%
Affine Model – recalibrated		3.0%	0.3%	0.8%	1.4%	1.8%

Evidence on the model's fit

Model matches regression results on the effects of bond supply.

- Coefficient on WAM holding 2Y yield constant:

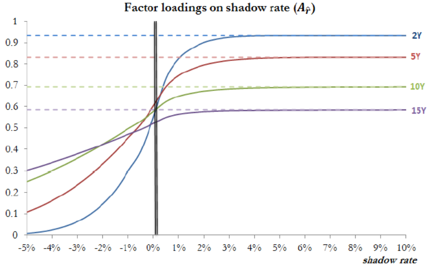
	Data		Model	
	above ELB	at ELB	$\widehat{r} = 5\%$	$\widehat{r} = -2\%$
5Y	0.10	0.00	0.06	0.03
10Y	0.19	0.05	0.14	0.10
15Y	0.23	0.11	0.19	0.15

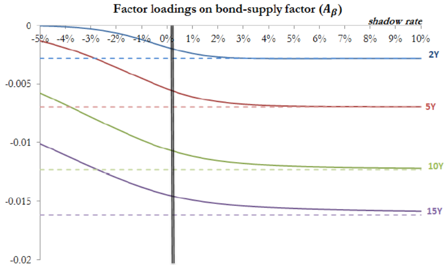
- Coefficient on 2Y yield holding WAM constant:

	Data		Model	
	above ELB	at ELB	$\widehat{r} = 5\%$	$\widehat{r} = -2\%$
5Y	0.90	1.9	0.90	2.0
10Y	0.79	2.3	0.70	2.5
15Y	0.75	2.2	0.64	2.4

Factor loadings in the shadow-rate model

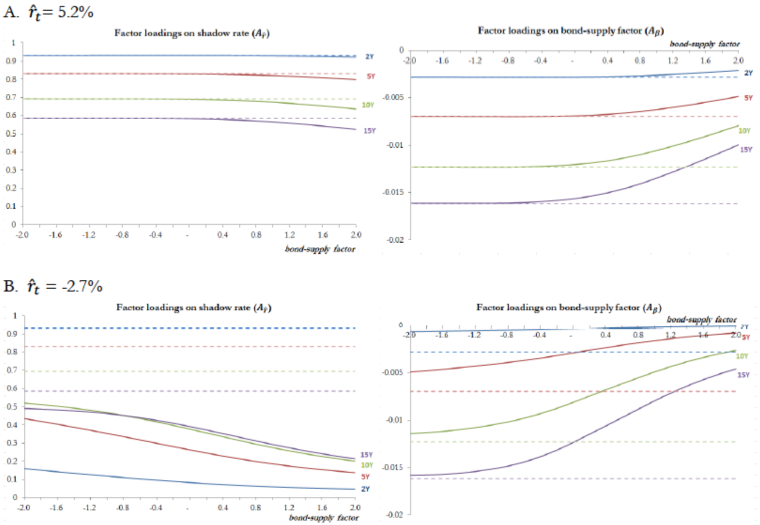
- In an affine model, factor loadings are constant.
- In the nonlinear model, they are state-dependent.





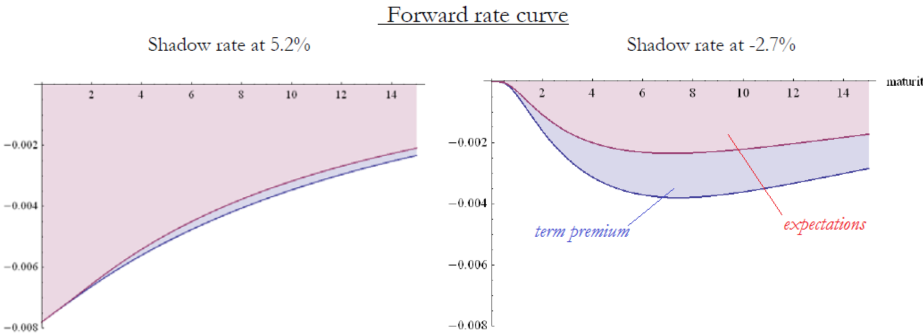
- The sensitivity to both factors is *quantitatively* attenuated by the ELB.
- The $\hat{r}_t$ loadings change *qualitatively*, reversing their order across maturities.

Factor loadings in the shadow-rate model



Effects of shadow-rate shock on yield curve components

Impact of a one-standard-deviation shock to $\hat{r}_t$ from different initial values:



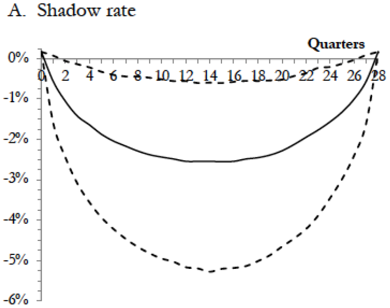
- At the ELB:
  - Overall effects are smaller.
  - Effects are increasing, not decreasing, across maturities.
  - Effects on the term premium are important.

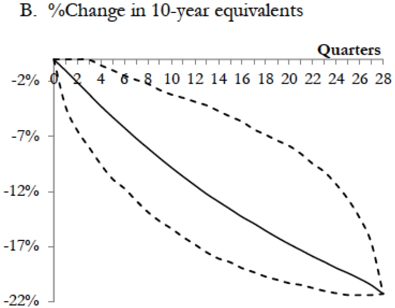
Assessing unconventional monetary policy

To study the effects of actual Fed policy in this model, I calculate shocks that correspond to what the Fed actually did:

- Shadow rate shocks - kept r_t at the ELB for 7 years.
- Fed balance sheet shocks - removed 21% of government-backed duration.
  - These are assumed to be less persistent than the β_t shocks above, but this makes little difference.

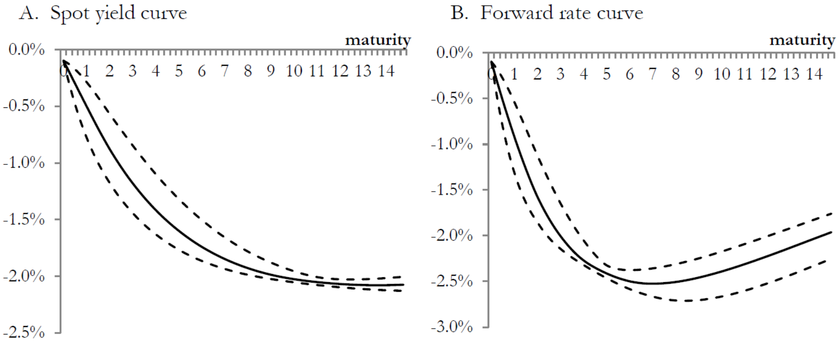
Consider a set of trajectories that are consistent with these observations:





Cumulative yield-curve responses in model sims

Adding up the yield-curve surprises (pseudo event study):



- Magnitude is roughly consistent with the cumulative effects of unconventional policy implied by event studies.
- Model captures the "hump shaped" forward-curve response noted by Rogers et al. (2014) and others.

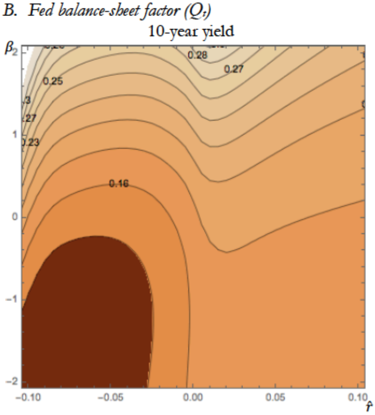
Decomposition of yields w/r/t unconventional policy shocks

	Shadow-rate shocks		Fed balance-sheet shocks		
Maturity [1]	Expectations component [2]	Term premium component [3]	Term premium component [4]	Interaction [5]	Total [6]
2 years	-59	-22	-13	7	-90
	(-82, -39)	(-25, -16)	(-14, -12)	(5, 8)	(-116, -63)
5 years	-90	-51	-30	12	-160
	(-106, -69)	(-52, -47)	(-31, -26)	(9, 14)	(-177, -135)
10 years	-102	-70	-47	12	-207
	(-109, 91)	(-76, -62)	(-50, -41)	(8, 16)	(-211, -199)
15 years	-98	-72	-57	10	-215
	(-100, -92)	(-82, -63)	(-60, -49)	(7, 14)	(-219, -210)

- Shadow-rate shocks account for over 75% of the effects of unconventional policy on long-term yields.
- About 1/3 of this effect comes from the effects on term premia through reduced volatility.

Relative efficacy of different tools

Size of β shock needed to equate to a -25bp $\widehat{r}$ shock:



Balance sheet is *relatively* more effective when shadow rate is negative and duration is high.

Conclusion

- Simple no-arbitrage model of bond portfolio choice w/shadow rate.
- Captures both forward guidance/signaling and duration channel of QE.
- At the ELB, things change dramatically:
  - Effects of both types of shocks are attenuated by the ELB.
  - Forward guidance has effects on term premia at the ELB that don't exist elsewhere.
- Consequently, the effects of unconventional monetary policy at the ELB may not be well described by
  - Empirical estimates from pre-ELB data
  - Theoretical models that assume linearity
- Simulations suggest that communications about future short rates were far more important for yields than was duration removal during the ELB period.